Stocastic Sec 5

Mean & Variance

⇒ **Discrete Random Variable**

The Complete description of a discrete random Variable requires specifying the possible Values that the random Variable may take and the probabilty associated with each Value (Probabilty distribution)

X	1	2	3	4	.
P(X)	✓	✓	✓	✓	. . .

⇒ **Mean:** (expectation, expected Value)

$E(X)$, μ_X , μ

First moment of the origin $= \sum_X X\, P(X)$

Secand moment of the origin $= \sum_X X^2 P(X)$

r^{th} // // // // $= \sum_X X^r P(X)$

⇒ **Variance:**

1^{st} moment about the mean $= E(X-\mu)$
$= \sum_X (X-\mu)\, P(X)$

2^{st} moment about the mean $= E(X-\mu)^2$
$= \sum_X (X-\mu)^2 P(X)$
= Varbnce
$= Var(X) = \sigma^2$

⇒ **Standard Deviation:** σ (S.D.)

$\sigma = \sqrt{Var(X)}$

Ex: prove that $Var(X) = E(X^2) - \mu^2$

$$Var(X) = \sum_X (X-\mu)^2 P(X)$$
$$= \sum_X (X^2 - 2\mu X + \mu^2)\, P(X)$$
$$= \sum_X X^2 P(X) - 2\mu \underbrace{\sum X P(X)}_{\mu} + \mu^2 \underbrace{\sum_X P(X)}_{1}$$
$$= E(X^2) - 2\mu^2 + \mu^2$$
$$= E(X^2) - \mu^2$$

⇒ **Continous Random Variable**

⇒ **Mean**

1^{st} moment about the origin $= E(X) = \int_{-\infty}^{\infty} X\, P(X)$

2^{st} // // // // $= E(X^2) = \int_{-\infty}^{\infty} X^2 P(X)$

⇒ **Variance**

1^{st} moment about the mean $= E(X-\mu) = \int_{-\infty}^{\infty} (X-\mu)\, P(X)$

2^{nd} // // // $= E(X-\mu)^2 = \int_{-\infty}^{\infty} (X-\mu)^2 P(X)$
= Variance

Sheet (4)

$\boxed{7}$ plot the graph of $F(X)$ for the Variable X with density

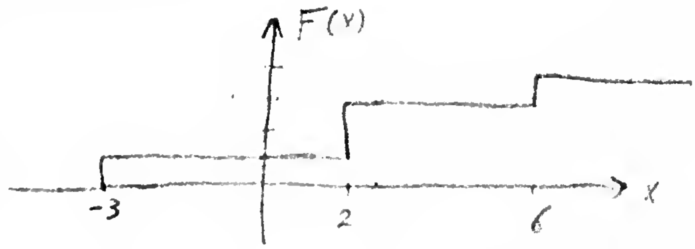
X	-3	2	6
P(X)	0.25	0.5	0.25

$F(-\infty) = 0$

$F(-3) = P(-3) = \frac{1}{4}$

$F(2) = P(2) + P(-3) = \frac{3}{4}$

$F(6) = 1$



Sheet [3]

[1] X equal the no. of Successful cures out of the Five

X	0	1	2	3	4	5
P(X)	0.002	0.029	0.132	0.309	0.36	0.168

Find μ, σ

$\mu = \sum_x x P(x) = 0(0.002) + 1.(0.029) + 2(0.132) + 3(0.309) + 4(0.36) + 5(0.168)$

$= 3.5$

$\sigma^2 = E(x^2) - \mu^2 = \sum_x x^2 P(x) - \mu^2$

$= 0(0.002) + 1.(0.029) + 2^2(0.132) + 3^2(0.309) + 4^2(0.36) + 5^2(0.168) - (3.5)^2$

$= (1.02)^2$

$\sigma = 1.02$

[2] iv) Find the expectation, Variance, S.D.

$$P(X) = \begin{cases} \frac{2}{25} x & 0 \le x \le 5 \\ 0 & \text{otherwise} \end{cases}$$

$\mu = \int_{-\infty}^{\infty} x P(x) dx = \int_0^5 \frac{2}{25} x^2 dx$

$= \frac{2}{25} \left[\frac{x^3}{3} \right]_0^5 = \frac{10}{3}$

$Var(x) = E(x^2) - \mu^2$

$= \int_{-\infty}^{\infty} x^2 P(x) dx - \mu^2$

$= \checkmark$

$\sigma = \sqrt{Var(x)} = \checkmark$